DAMAGE DETECTION FOR APPLICATIONS UNDERGOING AXIAL (MEMBRANE) RESPONSE

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ABSTRACT. This paper extends and applies recently reported damage identification methods, previously utilized for flexural vibrations only, to axial- and membrane-type vibrations. The methods are applied to an 8-DOF linear spring-mass system, which models a multi-degree-of-freedom axial or membrane system. The goal of the work is to detect damage (as indicated by reduction in stiffness of one or more of the elements) as well as to locate the damaged elements. Two damage detection methods were investigated: the 'change-in-flexibility' method and the 'damage-index' method. Both were found to successfully locate the damaged element(s) for 10-percent reduction in element stiffness. The 'change-in-flexibility' method indicated damage location even when only a limited number of lower modes were included.

NOMENCLATURE

a :Axial position along span
AE :Axial rigidity
EI :Flexural rigidity
g_{ik} :Axial rigidity ratio for i^{th} mode shape and k^{th} region
k_i :Stiffness of i^{th} element
ℓ :Span of structure
m_i :Mass of i^{th} element
n :Number of measured or calculated modes
U :Strain energy
u :In-plane displacement
w :Transverse displacement
x :Axial Coordinate
α_k :Damage index
δ_{ij} :Elements of change-in-flexibility matrix
λ_i :i^{th} eigenvalue
ω_i :i^{th} frequency of vibration
$[F]$:Flexibility matrix
$[K]$:Stiffness matrix
$[M]$:Mass matrix
$\{\psi_i\}$:i^{th} mode shape
$[\Psi]$:Mode shape matrix
$[\Delta F]$:Change-in-flexibility matrix
$[\Omega]$:Modal stiffness matrix
$\{\phi_i\}$:i^{th} mass-normalized mode shape
$[\Phi]$:Mass-normalized mode shape matrix
$[\ ]^T$:Transpose
$[\ ]^{-1}$:Inverse
$(\ )^*$:Property of damaged structure

1. INTRODUCTION

A wealth of damage identification algorithms exist for detecting damage as well as its location in structural and mechanical systems [1,2]. Often these methods are applied to structures, such as bridges, undergoing flexural vibrations. However, it appears that use of such methods for applications involving axial or membrane-type vibrations is limited. Damage detection in truss structures exhibiting bending, torsional, and axial modes is discussed in [3,4].

The purpose of this paper is to extend and apply recently reported damage identification methods, previously utilized for flexural vibrations only, to axial-type vibrations. Two such methods are investigated in detail: The 'change-in-flexibility' method and the 'damage index' method. The methods are applied to an 8-DOF linear spring-mass system, which models a multi-degree-of-freedom axial or membrane system. The goal of the work is to detect damage (as indicated by reduction in stiffness of one or more of the elements) as well as to locate the damaged element(s).

As a starting point, a complete set of natural frequencies and normal modes were determined numerically (MATLAB [5] was used in all studies reported herein) and then corresponding sets of modal properties were determined for eight simulated cases of "damage". Damage consisted of a 10 percent independent reduction of stiffness in each element. Not surprisingly, detection of damage by comparison of either natural frequency or mode shape changes between undamaged and damaged cases was generally unsuccessful. Further, no information could be gleaned on damage location using this primitive damage detection approach. More robust methods were then extended to this axial vibration problem.

The 'change-in-flexibility' method [6] is based upon differences in the flexibility of the structure before and after damage has occurred. For the flexural vibrations of beams, Pandey and Biswas show that damage could both be detected and located from just the first two (or three) measured modes of the structure. The method differs somewhat from the more familiar methods of damage detection based on changes in the modal parameters of a system (i.e., resonant frequencies, mode shapes, and modal damping). The authors are unaware of any application of the method to problems involving axial loading.

This "change-in-flexibility' method was applied to the 8-DOF spring-mass model and found to be particularly robust in both the detection of the presence of damage as well as the location of the damage. The location of damage is directly indicated by a step change in a flexibility indicator at the point of damage. Further, the *magnitude* of this indicator does not depend upon damage location, in contrast to earlier work on beams reported in [6].

Because in application there is a distinct possibility of damage at more than one of the elements, the case of simultaneous damage at more than one element location was investigated as well. The 'change-in-flexibility' method was found to isolate two separate damage locations.

The above investigation was performed numerically using a complete set of vibration modes. In practice, only a partial set of the lower modes may be available. The influence of including a partial set of modes was investigated for the case of damage at the center element. For 10-percent reduction in element stiffness, it was found that the location of damage was clearly indicated using only the first mode. Further, for the particular 8-DOF example evaluated, a reasonable estimate of the magnitude of the damage indicator was provided using the first vibration mode only. Inclusion of the first three modes was sufficient to estimate the damage indicator (flexibility change) within a few percent.

The modification of the Damage-Index method [7] to treat axial, as opposed to bending, problems is also presented. Using the same 8-DOF system with and without damage, the location of damage was readily determined for the case in which complete sets of normal modes were used.

2. FREE VIBRATION MODEL

Consider the linear 8-DOF spring-mass system shown in **Fig. 1**, for which free vibration response is described by solution of the generalized eigenvalue problem,

$$[K]\{\psi_i\} = \lambda_i [M]\{\psi_i\} \tag{1}$$

where $\lambda_i = \omega_i^2$. Here [M] denotes the diagonal mass matrix, [K] is the (symmetric) stiffness matrix, λ_i is the *ith* eigenvalue (square of the *ith* natural frequency) and $\{\psi_i\}$ denotes the *ith* eigenvector (*ith* mode shape). If [M] is the identity matrix, Eq. (1) reverts to the standard eigenvalue equation [8].

By inspection of Fig. 1, the mass matrix is

$$[M] = \begin{bmatrix} m_1 & & & & 0 \\ & m_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ 0 & & & & m_8 \end{bmatrix} \tag{2}$$

The stiffness matrix, [K], can be readily determined to be the following banded 8 x 8 symmetric matrix:

$$[K] = \begin{bmatrix} (k_1+k_2) & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & (k_2+k_3) & -k_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & (k_3+k_4) & -k_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_4 & (k_4+k_5) & -k_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_5 & (k_5+k_6) & -k_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_6 & (k_6+k_7) & -k_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_7 & (k_7+k_8) & -K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_8 & k_8 \end{bmatrix} \tag{3}$$

2.1 Baseline Case

For illustrative purposes, consider a baseline 8-DOF system of unit masses and stiffnesses:

$$\begin{aligned} m_i &= 1 \ for \ i = 1,8 \\ k_i &= 1 \ for \ i = 1,8 \end{aligned} \tag{4}$$

where m_i and k_i are the mass and stiffness values defined in **Fig. 1**.

The eight eigenvectors and corresponding eigenvalues for the generalized eigenvalue problem are readily determined numerically [5]. The natural frequencies can be written as [9]:

$$\omega_i = \frac{1}{\pi} sin \left[\frac{(2i-1)}{17}\pi\right], \ i = 1, 2, ..., 8 \tag{5}$$

2.2 Introduction of "Damage"

Now each of the eight stiffnesses, k_i, is individually reduced 10-percent to a value of 0.9 to simulate damage i.e., there are 8 separate cases of damage considered. A plot of natural frequency as a function of mode number for the baseline system and the eight perturbed systems is shown in **Fig. 2**. Clearly, identification of damage by changes in natural frequency is not feasible, as a 10-percent stiffness change at one location results in almost negligible natural frequency changes for normal modes.

The effects of stiffness perturbations on mode shape changes are illustrated in **Fig. 3**. **Fig. 3a** shows the normalized modal displacement of the first mode for the baseline unperturbed case as well as the eight perturbed (10-percent reduced stiffness) cases. **Fig. 3b** is the corresponding set of plots for mode 8. Generally, it was found that the four highest modes do seem somewhat more sensitive to stiffness perturbations. In some cases, the percentage change can be quite substantial for individual degrees of freedom. Unfortunately, modal shapes are more difficult to accurately obtain than frequencies. Moreover, no information can be gleaned from these natural frequency or modal shape changes regarding detection of the location of the damage.

In the following two sections, available damage ID methods are utilized to determine their ability to more robustly isolate the above damage scenarios.

3. APPLICATION OF THE 'CHANGE-IN-FLEXIBILITY' METHOD

3.1 Brief Summary of Method

As explained in [1], the method consists of the following: For the undamaged structure, the flexibility matrix, [F], is derived from the modal data a follows:

$$[F^*] = [\Phi][\Omega]^{-1}[\Phi]^T \approx \sum_{i=1}^{n} \frac{1}{(\omega_i)^2}\{\phi_i\}\{\phi_i\}^T , \qquad (6)$$

where $\{\phi_i\}$ = the *ith* mass normalized mode shape,

$[\Phi]$ = the mass-normalized mode shape matrix = $[\phi_1,\ \phi_2,\ ...\phi_n]$,

ω_i = the *ith* modal frequency,

$[\Omega]$ = The modal stiffness matrix = diag. (ω_i^2), and

n = the number of measured or calculated modes.

The approximation in Eq. 6 comes from the fact that typically the number of modes identified is less than the number of degrees of freedom needed to accurately represent the motion of the structure. Similarly, for the damaged structure

$$[F^*] = [\Phi^*][\Omega^*]^{-1}[\Phi^*]^T \approx \sum_{i=1}^{n} \frac{1}{(\omega_i^*)^2}\{\phi_i^*\}\{\phi_i^*\}^T , \qquad (7)$$

where the asterisks signify properties of the damaged structure. From the pre- and post- damage flexibility matrices, a measure of the flexibility change caused by the damage can be obtained from the difference of the respective matrices, i.e.,

$$[\Delta F] = [F] - [F^*], \qquad (8)$$

where ΔF represents the change-in-flexibility matrix. Now, for each column of matrix ΔF let $\bar{\delta}_j$ be the absolute maximum value of the element in the jth column. Hence,

$$\bar{\delta}_j = max|\delta_{ij}|,\ i = 1,\ ...,\ n \qquad (9)$$

where $\bar{\delta}_{ij}$ are elements of matrix ΔF. $\bar{\delta}_j$ is taken to be a measure of the flexibility change at each measurement location. The column of the flexibility matrix corresponding to the largest $\bar{\delta}_j$ is indicative of the degree of freedom where damage is located.

3.2 Application of the Method

Imagine now that the modal frequencies and mode shapes, determined in Section 2 are actually experimental data. In this case, then, all mode shapes and modal frequencies are known precisely. Proceeding from this starting point, the 'change-in-flexibility' method is applied.

First, each mode shape of the baseline (undamaged) case is mass-normalized [10]. (Experimentally, this would be accomplished by making a driving point response measurement). The procedure for determining the mass-normalized ("weighted") modal matrix [Φ] is to first determine the generalized masses for each mode. This is done by first forming the matrix product $[\Psi]^T$ [M] [Ψ] where [M] is the original mass matrix, and [Ψ] denotes the (unweighted) modal matrix calculated earlier. The diagonal terms of the matrix product denote the generalized mass, M_i. Then each of the columns of the modal matrix [Ψ] is divided by the square root of the appropriate generalized mass, M_i, resulting in the mass-normalized modal matrix [Φ].

Next, the baseline flexibility matrix, [FB] is determined from

$$[FB] = [\Phi][\Omega]^{-1}[\Phi]^T \qquad (10)$$

and is shown in **Table 1.**

Table 1. Baseline Flexibility Matrix, FB

$$\begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 \\ 1.0 & 2.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 5.0 & 5.0 & 5.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 6.0 & 6.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 7.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 \end{bmatrix}$$

The inverse of his matrix is of course identical to the stiffness matrix calculated from Eq. (3), as a full set of eigenvectors and eigenvalues were used in generating [FB].

Referring to **Table 1**, it is seen that flexibility values, at least along the main diagonal, increase as one moves away from the fixed end. This is in agreement with the definition of flexibility coefficients. For example, a unit force applied to node 8 causes node 8 to displace 8 times as far as would node 1 when subjected to a unit force as the springs are in series.

Now the flexibility of each of the 8 perturbed systems (stiffness reduction of 10-percent) is calculated. Then the matrices of the differences between the original and the perturbed flexibility matrices are calculated (Eq. 8) and $\bar{\delta}_j$, Eq. 9, is determined. Normalized results are presented in **Fig. 4** for 10-percent damage, respectively, in the first, second and third elements. It is apparent that the detection of damage location is indeed possible for axial-loading applications and takes on a somewhat different form than for the beam bending examples discussed in [6]. For the situation in **Fig. 4**, damage is present in the element for which nonzero differences in flexibility first appear. Further, the magnitude of the damage indicator is independent of damage location, at least for the particular example considered.

An advantage of the method, when applied to this axially loaded example, is that the location of damage is directly indicated by a step change at the point of damage. The method does, however, require mass-normalized mode shapes.

3.3 Damage at Multiple Locations

The next issue is that of damage at multiple locations. The case of damage at DOF 3 and DOF 6 (a 10-percent reduction in stiffness at each location) is shown in **Fig. 5**. Again, the method isolates both the presence and location of damage. Additional damage locations are indicated by subsequent step changes in the damage indicator.

3.4 Influence of Number of Modes Used

For axial (membrane) problems, the influence of the number of lower modes utilized is illustrated in **Fig. 6** for the case of "damage" (10-percent reduction in stiffness of k_4). In **Fig. 6**, it is seen that the presence and location of damage is readily estimated using only the first mode "data" for determining the change in flexibility.

4. APPLICATION OF THE 'DAMAGE-INDEX' METHOD

The damage index method, developed by Stubbs and Kim [7], locates damage in structures undergoing bending vibration given their characteristic mode shapes measured before and after damage. For structures undergoing bending, it was found that only a few modes are required to obtain reliable results.

The method is based on an examination of modal strain energies in undamaged and damaged beams. It is straightforward to modify the method to account for axial, as opposed to bending, vibrations. Following the derivation for beam bending by Cornwell, et al. [11], the bending strain energy for a Bernoulli-Euler beam is

$$U = \frac{1}{2}\int_0^{\ell} EI\left(\frac{d^2w}{dx^2}\right)^2 dx \tag{11}$$

where EI is flexural rigidity ℓ denotes beam length, w is transverse displacement, and x is the coordinate along the span of the beam. The corresponding energy expression for axial vibrations can be written

$$U = \frac{1}{2}\int_0^{\ell} AE\left(\frac{du}{dx}\right)^2 dx \tag{12}$$

where $u(x)$ denotes the in-plane (axial) displacement field, and AE denotes the axial rigidity.

Making the appropriate modifications to the derivation by Cornwell, et. al. [11], it is readily shown that the change in axial rigidity (as opposed to bending rigidity) at the k^{th} location in the structure for the **ith** mode is given by

$$\frac{g_{ik}^*}{g_{ik}} = \frac{\int_{a_k}^{a_{k+1}}\left(\frac{d\psi_i^*}{dx}\right)^2 dx \Big/ \int_0^{\ell}\left(\frac{d\psi_i^*}{dx}\right)^2 dx}{\int_{a_k}^{a_{k+1}}\left(\frac{d\psi_i}{dx}\right)^2 dx \Big/ \int_0^{\ell}\left(\frac{d\psi_i}{dx}\right)^2 dx} \tag{13}$$

where ψ_i denotes the *ith* normal mode shape and $(\,)^*$ denotes the case of damage. The above expression is an index of the change in axial rigidity from undamaged to damaged structures. $a_k \le x \le a_{k+1}$ denotes the interval along the span, ℓ, of the k^{th} region. If the above equation is reapplied for each sub-region (i.e., element) along the span, a measure of axial rigidity change along the span *for the ith normal mode* is produced.

In order to use all measured modes, n, the damage index, α_k, for the k^{th} subregion is defined as

$$\alpha_k = \frac{\sum_{i=1}^{n} g_{ik}^*}{\sum_{i=1}^{n} g_{ik}} \tag{14}$$

The above procedure was implemented for the 8-DOF system introduced earlier (see **Fig. 1**). Results are shown in **Fig. 7** for the case in which "damage" was introduced in the third sub-region (element). Again, the damage consisted of a 10-percent reduction in stiffness at that location. The location of damage at sub-region 3 is readily apparent in **Fig. 7**.

It is important to note that **Fig. 7** is constructed utilizing the complete set of all eight (damaged and undamaged) modes. Future work will include the evaluation of the method for a reduced set of lower modes. In addition, the possibility of multiple damage locations will be investigated with the method. An advantage of the method is that mass-normalized mode shapes are not required.

5. SUMMARY AND CONCLUSIONS

Two recently reported structural damage identification methods, previously developed for structures undergoing flexural vibrations, were applied to a spring-mass system undergoing axial response. Both the 'change-in-flexibility' method and a modified form of the 'damage-index' method were successful in detecting damage and in locating damaged elements for 10-percent reduction in element stiffness.

To date, the applications of vibration-based damage identification methods have been primarily focused on relatively simple structures such as beams and plates [1]. By developing a library of "damage elements" the authors believe that it may be possible to construct a methodology of vibration-based damage identification, analogous to finite element discretization, that can be more readily applied to general structural and mechanical systems. The damage index method is particularly well suited for such development. The development of a truss or membrane element presented in this work provides the foundation for one element of such a code. Previous work of Stubbs and Kim [5] and Cornwell [9], provide the development of two additional elements (beams and plates) for such a code. Further development will be required for shell and continuum damage elements. Consideration of the fact that the vibration responses associated with internal degrees of freedom (those nodes not accessible to sensors mounted on the surface of the structure) are extremely difficult to measure must be addressed in the development of such a general purpose code. Also, difficulties associated with measuring rotational degrees of freedom will have to be addressed.

The robust damage detection capabilities of the axial (or membrane) element developed in this study are attributed to direct measure of mode-shape difference quantities needed in the application of the methods. Most commonly, translational degrees of freedom are measured in vibration testing. For axial or membrane elements, measurements of the translational degrees of freedom at either end of the member provide a complete description of the element's flexibility or stored strain energy. In contrast, to obtain flexibility or strain energy quantities associated with beam and plate bending, the curvature must be interpolated from translation measurements. The authors speculate that this interpolation process and the subsequent differentiation of interpolated quantities when the damage index method is used leads to errors that can often make the assessment of damage ambiguous [1].

Future work will include an evaluation of the sensitivity of the methods to measurement errors which would be present in actual signals. The viability of the methods for reduced measurement locations is also a subject of future investigation.

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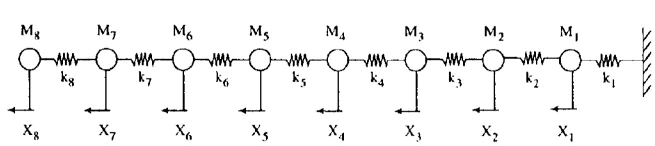
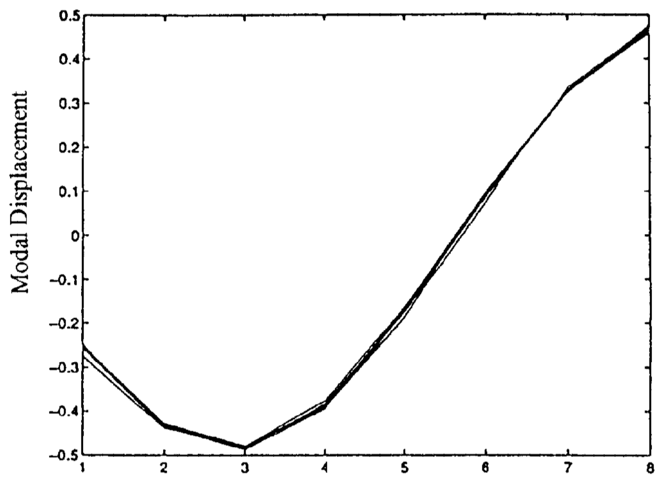


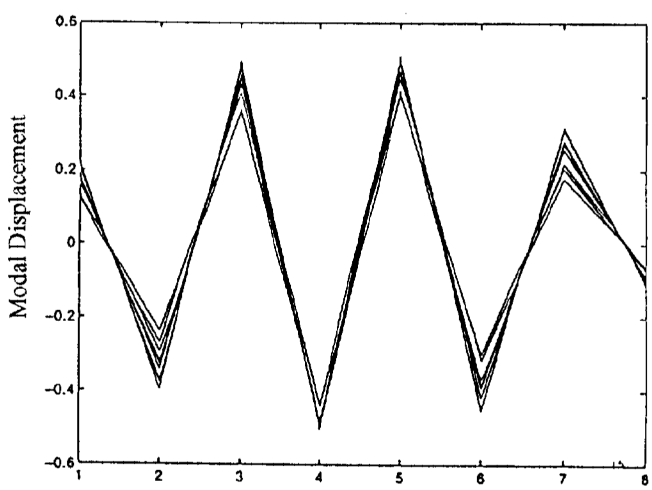
Figure 1. Eight degree-of-freedom spring-mass system.

Figure 2. Comparison of natural frequencies for the 8 modes when stiffness of each spring is independently reduced by 10-percent. Baseline (unperturbed) case also shown.



DOF Number

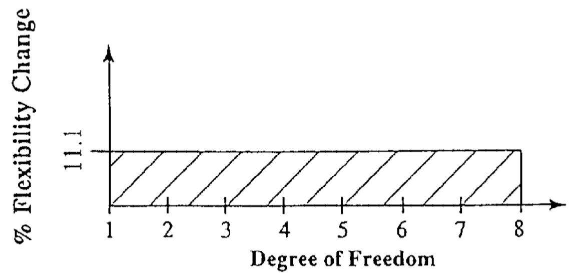
a. 1st Mode Eigenvectors



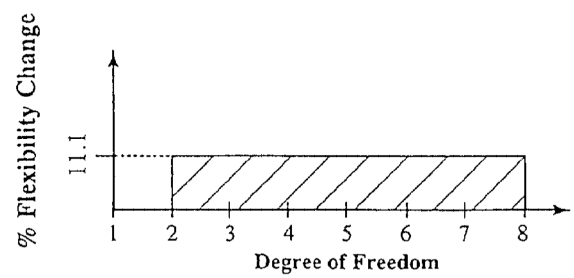
DOF Number

b. 8th Mode Eigenvectors

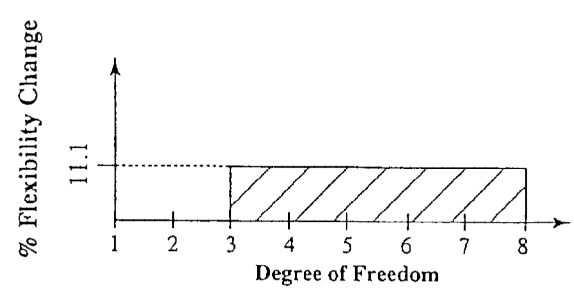
Figure 3. Comparison of unperturbed and all perturbed (reduced stiffness) cases for lowest and highest modes.



a. Damage in first element



b. Damage in second element



c. Damage in third element

Figure 4. Damage detection using 'change-in-flexibility' method.

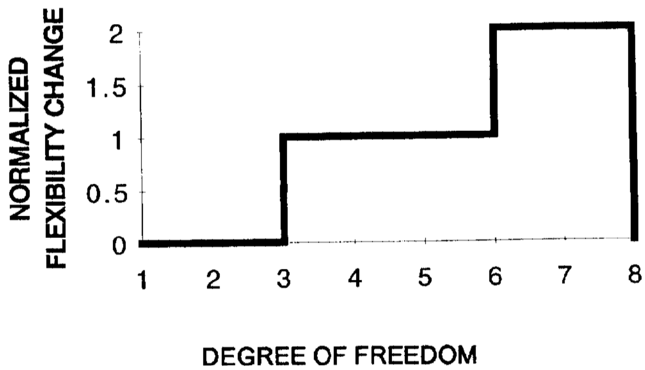


Figure 5. Simultaneous damage at two locations (10%) reduction in k_3 and k_6.

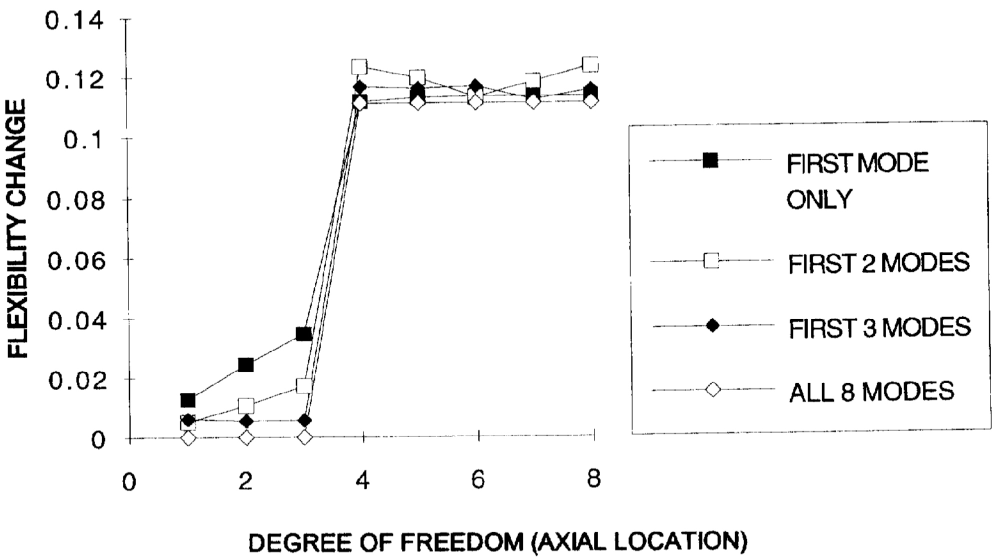


Figure 6. Influence of number of modes included for 'change-in-flexibility' method.

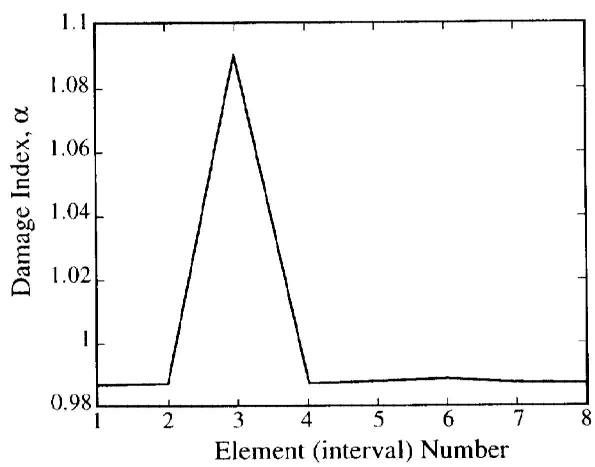


Figure 7. Detection of damage at the third interval using the 'damage-index' method.